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MAGNETIC HYPERSONIC RAREFIED FLOW AT THE STAGNATION POINT OF A BLUNT BODY WITH SLIP AND MASS TRANSFER

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Abstract—The hypersonic viscous flow of a slightly rarefied electrically conducting gas with variable properties in the stagnation region of a blunt body with an applied magnetic field, slip and mass transfer (suction or injection) has been studied. The similarity solutions of the boundary-layer equations have been obtained numerically using the method of quasilinearization. The results indicate the extent to which the magnetic field, slip and mass-transfer parameters etc. affect the heat-transfer rate.

NOMENCLATURE

- B_0, applied magnetic field;
- C, density-viscosity product ratio, $\rho\mu/\rho_e\mu_e$;
- C_f, skin friction coefficient;
- C_p, specific heat at constant pressure;
- f, nondimensional stream function;
- f_w, mass-transfer parameter, $-(\rho v)_w(2\xi)^{\frac{1}{2}}/\rho_e\mu_e u_e r_0$;
- f', nondimensional velocity, u/u_e;
- f'', nondimensional shear stress function;
- g, nondimensional total enthalpy, I/I_e;
- g_w, cooling parameter, I_w/I_e;
- g', nondimensional heat-transfer function;
- I, total enthalpy;
- k_1, thermal conductivity;
- l, molecular mean free path of the gas;
- M, magnetic parameter;
- n, constant;
- Nu, Nusselt number;
- Q, R, S, functions defined by equation (13);
- Pr, Prandtl number;
- $-\dot{q}_w$, heat-transfer rate;
- r_0, distance from the axis of the body of revolution;
- R_x, Reynolds number;
- u, v, velocity components along and perpendicular to the surface of the body;
- x, y, distances along and perpendicular to the surface of the body.

Greek symbols

- α, α', reflection and thermal accommodation coefficients respectively;
- β, pressure gradient parameter;
- γ, ratio of specific heats;
- η, transformed coordinates;
- λ, λ_1, slip parameters;
- μ, coefficient of viscosity;
- ν, kinematic viscosity;
- ρ, density;
- σ, electrical conductivity;
- τ, shear stress;
- χ, arbitrary constant;
- ψ, dimensional stream function;
- ω, exponent in the power-law variation of viscosity.

Superscripts

- $'$, denotes differentiation with respect to η;
- k, denotes iterative index.

Subscripts

- c, denotes complementary solution;
- e, denotes conditions in stream external to the boundary layer;
- p, denotes particular solution;
- w, denotes conditions at the surface $y = \eta = 0$;
- ∞, denotes asymptotic function $\eta \to \infty$.

1. INTRODUCTION

RECENT successful flights of manned satellites has opened up a new regime in the hypersonic flight spectrum. The design of space vehicles requires a fundamental knowledge of the aerodynamic characteristics of blunt bodies in rarefied and free molecule flow regions at high flight Mach numbers. Therefore, the study of rarefied gas flows over blunt bodies is important in practical hypersonic flights at high altitudes. In hypersonic flow over blunt bodies, the boundary-layer thickness as well as the temperature of the body increases with Mach number. The high stagnation temperature encountered in re-entry vehicles renders the air sufficiently ionized behind the bow shock so that it may be considered as electrically conducting. Under these circumstances, the presence of a magnetic field helps to reduce the heat-transfer rates. Suction helps to reduce the interaction of the boundary layer with the external flow while surface injection further alleviates the heat transfer to the body.

The effect upon the axisymmetric stagnation point heat transfer produced by the interaction of a magnetic field with an electrically conducting fluid in a compressible boundary layer with constant properties (i.e. density–viscosity product $\rho\mu$ and the Prandtl number Pr are constants), in the absence of mass transfer and slip has been studied by Meyer [1], Lykoudis [2] and Bush [3]. Bush obtained the solution by means of a finite difference technique. He was unsuccessful with the method of forward integration owing to the serious lack of information available as to what values to assume for the derivatives at the initial boundary. Nath [4] solved the same problem with variable properties (i.e. $\rho \propto h^{-1}$, $\mu \propto h^{\omega}$, $Pr = 0.72$), but only for small values of the magnetic parameter. Chen [5] has investigated the effect of a small magnetic field and wall temperature on the flow field at the stagnation point of a blunt nosed body in low Reynolds number hypersonic flow using modified Rankine–Hugoniat shock relations in conjunction with two-layer formulation proposed by Cheng [6–7].

Our objective in this paper is to study the hypersonic flow of a viscous, slightly rarefied, electrically conducting gas with variable properties, in the neighbourhood of the stagnation point of a blunt body. There is an applied magnetic field, together with slip and mass transfer. The similarity solutions of the boundary layer equations with variable properties have been obtained numerically using the method of quasilinearization [8–12]. The results exhibit the effect of these parameters on the heat transfer rate.

2. GOVERNING EQUATIONS

We consider a model gas (variable gas properties were modelled realistically by taking $\rho \propto h^{-1}$, $\mu \propto h^{\omega}$, $Pr = 0.72$) which is slightly rarefied and electrically conducting. We have considered the Prandtl number as constant because in most atmospheric flight problems its variation in boundary layer is quite small [13]. The electrical conductivity has been assumed to vary as $\sigma \propto h^n$. The flow is steady, laminar, and hypersonic in the neighbourhood of the stagnation point of an axisymmetric insulating porous blunt body. An applied radial magnetic field B_0 is imposed on the boundary layer. The induced magnetic field is negligible in comparison with the applied field. Neglecting the Hall effect, the governing boundary-layer equations in dimensionless form under similarity considerations, valid for the stagnation region are [3–4]

$$(Cf'')' + ff'' + \beta[g\{1 + M(1 - g^n f')\} - f'^2] = 0 \quad (1)$$

$$(Cg')' + Prfg' = 0 \quad (2)$$

with $\rho_e/\rho = g$, $C = \rho\mu/\rho_e\mu_e = g^{\omega-1}$, $\sigma/\sigma_e = g^n$. The appropriate boundary conditions taking into account the effect of first order slip velocity and temperature jump can be expressed as [14–16]

$$\begin{aligned} &f(0) = f_w, \quad f'(0) = \lambda f''(0), \quad f'(\infty) \to 1 \\ &g(0) = g_w + \lambda_1 g'(0), \quad g(\infty) \to 1. \end{aligned} \quad (3)$$

In the above equations, the independent similarity variable η is defined by

$$\begin{aligned} \eta &= \frac{u_e}{(2\xi)^{\frac{1}{2}}} \int_0^y r_0 \rho \, dy, \\ \xi &= \int_0^x \rho_e \mu_e u_e r_0^2 \, dx. \end{aligned} \quad (4)$$

The dependent variable is the modified stream function $f(\eta)$ defined by

$$f(\eta) = \psi/(2\xi)^{\frac{1}{2}}. \quad (5)$$

Here f_w is the mass-transfer parameter. $f_w > 0$ for suction and $f_w < 0$ for injection. The slip parameters λ and λ_1 can be expressed as

$$\begin{aligned} \lambda &= [(2-\alpha)/\alpha][lr_0 \rho u_e/(2\xi)^{\frac{1}{2}}] \\ \lambda_1 &= [(2-\alpha')/\alpha'][2\gamma l/(\gamma+1)Pr][\rho r_0 u_e/(2\xi)^{\frac{1}{2}}]. \end{aligned} \quad (6)$$

From [15] we find that λ and λ_1 are of the same order of magnitude. Hence we can take $\lambda \sim \lambda_1$.

It is to be noted that $\omega = 0.7$ corresponds closely to conventional wind tunnel conditions, while $\omega = 0.5$ represents conditions encountered in hypersonic flight [17]. The value $\omega = 1$ represents the familiar constant density–viscosity product simplification ($C = 1$). In the literature [3, 18], the value of n has been taken to be between 1 and 10 without mentioning which particular value of n approximately represents the conditions encountered in hypersonic flight. We have also assumed that the injected fluid has the same property as the main stream fluid.

The quasi-linear version of equations (1) and (2) are

$$\begin{aligned} &{}^{(k+1)}f''' - [(1-\omega)^{(k)}g'^{(k)}g^{-1} - {}^{(k)}f^{(k)}g^{1-\omega}]^{(k+1)}f'' \\ &- [\beta^{(k)}g^{1-\omega}\{M^{(k)}g^{n+1} + 2^{(k)}f'\}]^{(k+1)}f' \\ &+ [{}^{(k)}f''^{(k)}g^{1-\omega}]^{(k+1)}f - (1-\omega)^{(k)}f''^{(k)}g^{-1\,(k+1)}g' \\ &+ [(1-\omega)^{(k)}f''^{(k)}g'^{(k)}g^{-2} + (1-\omega)^{(k)}f^{(k)}f''^{(k)}g^{-\omega} \\ &+ \beta\{(1+M)(2-\omega)^{(k)}g^{1-\omega} - M(2+n-\omega) \\ &{}^{(k)}g^{1+n-\omega(k)}f' - (1-\omega)^{(k)}g^{-\omega(k)}f'^2\}]^{(k+1)}g \\ &= (2-\omega)^{(k)}f^{(k)}f''^{(k)}g^{1-\omega} + \beta[(1+M)(1-\omega)^{(k)}g^{2-\omega} \\ &- (2-\omega)^{(k)}g^{1-\omega(k)}f'^2 \\ &- (2+n-\omega)M^{(k)}g^{2+n-\omega(k)}f'] \end{aligned} \quad (7)$$

$$\begin{aligned} &{}^{(k+1)}g'' - [2(1-\omega)^{(k)}g'^{(k)}g^{-1} - Pr^{(k)}f^{(k)}g^{1-\omega}]^{(k+1)}g' \\ &+ (1-\omega)[{}^{(k)}g'^{2\,(k)}g^{-2} + Pr^{(k)}f^{(k)}g'^{(k)}g^{-\omega}]^{(k+1)}g \\ &+ Pr^{(k)}g'^{(k)}g^{1-\omega(k+1)}f \\ &= (2-\omega)Pr^{(k)}f^{(k)}g'^{(k)}g^{1-\omega}. \end{aligned} \quad (8)$$

The boundary conditions are

$$\begin{aligned} &{}^{(k+1)}f(0) = f_w, \; {}^{(k+1)}f'(0) = \lambda^{(k+1)}f''(0) \\ &{}^{(k+1)}f'(\infty) = 1, \; {}^{(k+1)}g(0) = g_w + \lambda^{(k+1)}g'(0), \\ &{}^{(k+1)}g(\infty) = 1. \end{aligned} \quad (9)$$

It is assumed that functions with iteration index (k) are known and that functions with iteration index $(k+1)$ are to be determined.

3. ASYMPTOTIC SOLUTIONS

Since $\beta = 0.5$ in the present problem, it follows from [10] that there is no non-uniqueness problem. Hence, the boundary conditions at $\eta \to \infty$ can be imposed at a large but finite value of η, say $\eta = \eta^*$ such that $f' = g = 1$ with η^* adjusted so that f'' and g' are very small at $\eta = \eta^*$. However, it is worthwhile to obtain more accurate specification of boundary conditions from the asymptotic solutions to the describing equations [10–12]. We follow the procedure given in [10–12].

The analysis starts with the quasilinear form of the governing equations and considers the kth iterates to be the asymptotic solutions. This corresponds to linearizing the equations about conditions at $\eta \to \infty$, i.e. we take [10–12]

$$^{(k)}f = \eta - \chi,\ ^{(k)}f' = {}^{(k)}g = 1,\ ^{(k)}f'' = {}^{(k)}g' = 0. \quad (10)$$

Using these values in equations (7) and (8) and following the procedure outlined in [10–12] for obtaining the asymptotic solutions, we get

$$^{(k+1)}f''^* + Q[^{(k+1)}f'^* - 1] + R[^{(k+1)}g^* - 1] = 0 \quad (11)$$

$$^{(k+1)}g'^* + S[^{(k+1)}g^* - 1] = 0 \quad (12)$$

where

$$Q = [1 + \{\beta(M+2)+1\}/^{(k)}f^{*2}]^{(k)}f^*,$$
$$R = \beta(Mn-1)/Pr^{(k)}f^*,$$
$$S = Pr[1 + 1/Pr^{(k)}f^{*2}]^{(k)}f^* \quad (13)$$

and f^*, g^* and their derivatives are the values of f, g and their derivatives at $\eta = \eta^*$, where η^* is some suitably chosen matching point. The equations provide the desired boundary conditions replacing those given at infinity, i.e. $f'(\infty) = g(\infty) = 1$.

4. SOLUTIONS OF LINEAR EQUATIONS

Assuming a kth approximation for the functions f, g, η^* and their derivatives, the linear equations (7) and (8) are integrated in the range $0 \leqslant \eta \leqslant {}^{(k)}\eta^*$, so as to obtain numerically two complementary solutions and one particular solution. We assume that [11]

$$^{(k+1)}f = {}^{(k+1)}A_1\,{}^{(k+1)}f_{1,c} + {}^{(k+1)}A_2\,{}^{(k+1)}f_{2,c} + {}^{(k+1)}f_p$$
$$^{(k+1)}g = {}^{(k+1)}A_1\,{}^{(k+1)}g_{1,c} + {}^{(k+1)}A_2\,{}^{(k+1)}g_{2,c} + {}^{(k+1)}g_p \quad (14)$$

with similar expressions for their derivatives. Here $^{(k+1)}f_{i,c}$ and $^{(k+1)}g_{i,c}$ $(i = 1, 2)$ denote the complementary solutions and $^{(k+1)}f_p$ and $^{(k+1)}g_p$ denote the particular solutions of equations (7) and (8) satisfying the following initial conditions:

$$^{(k+1)}f_{1,c} = 0,\ ^{(k+1)}f'_{1,c}(0) = \lambda,\ ^{(k+1)}f''_{1,c}(0) = 1 \quad (15)$$

$$^{(k+1)}g_{1,c}(0) = 0,\ ^{(k+1)}g'_{1,c}(0) = 0$$
$$^{(k+1)}f_{2,c}(0) = {}^{(k+1)}f'_{2,c}(0) = {}^{(k+1)}f''_{2,c}(0) = 0$$
$$^{(k+1)}g_{2,c}(0) = \lambda,\ ^{(k+1)}g'_{2,c}(0) = 1 \quad (16)$$

$$^{(k+1)}f_p(0) = f_w,\ ^{(k+1)}f'_p(0) = {}^{(k+1)}f''_p(0) = 0$$
$$^{(k+1)}g_p(0) = g_w,\ ^{(k+1)}g'_p(0) = 0. \quad (17)$$

The constants $^{(k+1)}A_1$ and $^{(k+1)}A_2$ are determined from the conditions at $\eta = \eta^*$. They satisfy the algebraic equations

$$[^{(k+1)}f''^*_{1,c} + Q^{(k+1)}f'^*_{1,c} + R^{(k+1)}g^*_{1,c}]^{(k+1)}A_1 + [^{(k+1)}f''^*_{2,c} + Q^{(k+1)}f'^*_{2,c} + R^{(k+1)}g^*_{2,c}]^{(k+1)}A_2 + [^{(k+1)}f''^*_p + Q\{^{(k+1)}f'^*_p - 1\} + R\{^{(k+1)}g^*_p - 1\}] = 0 \quad (18)$$

$$[^{(k+1)}g'^*_{1,c} + S^{(k+1)}g^*_{1,c}]^{(k+1)}A_1 + [^{(k+1)}g'^*_{2,c} + S^{(k+1)}g^*_{2,c}]^{(k+1)}A_2 + [^{(k+1)}g'^*_p + S\{^{(k+1)}g^*_p - 1\}] = 0. \quad (19)$$

The initial guesses for the kth approximation (satisfying the boundary conditions exactly) have been taken as

$$^{(k)}f = f_w + \eta + [\exp(-\eta) - 1]/(1+\lambda)$$
$$^{(k)}f' = 1 - \exp(-\eta)/(1+\lambda),\ ^{(k)}f'' = \exp(-\eta)/(1+\lambda)$$
$$^{(k)}g = 1 - (1-g_w)\exp(-\eta)/(1+\lambda),$$
$$^{(k)}g' = (1-g_w)\exp(-\eta)/(1+\lambda). \quad (20)$$

5. SKIN FRICTION AND HEAT TRANSFER

The skin friction coefficient C_f at the stagnation point of an axisymmetric body can be written as [3–4]

$$C_f(R_x)^{\frac{1}{2}} = 2^{\frac{1}{2}}f''(0)/(g_w)^{1-\omega} \quad (21)$$

where

$$C_f = 2\tau/\rho_e u_e^2,\ R_x = (du_e/dx)x^2/\nu_e. \quad (22)$$

The ratio of the skin friction coefficients with and without magnetic field is given by [3–4],

$$C_f(R_x)^{\frac{1}{2}}/[C_f(R_x)^{\frac{1}{2}}]_{M=0} = f''(0)/[f''(0)]_{M=0} \quad (23)$$

where

$$R_x/[R_x]_{M=0} = (du_e/dx)/[du_e/dx]_{M=0}. \quad (24$$

The ratio of the velocity gradients for various values of M can be obtained from [19].

Similarly, the Nusselt number Nu can be expressed as [3–4]

$$Nu(R_x)^{-\frac{1}{2}} = 2^{\frac{1}{2}}g'(0)/(1-g_w)g_w \quad (25)$$

where

$$Nu = (-\dot{q}_w C_p x)/k_1(I_e - I_w). \quad (26)$$

The ratio of the Nusselt number in the presence and in the absence of the magnetic field is [3–4]

$$Nu(R_x)^{-\frac{1}{2}}/[Nu(R_x)^{-\frac{1}{2}}]_{M=0} = g'(0)/[g'(0)]_{M=0}. \quad (27)$$

6. RESULTS AND DISCUSSION

The governing set of equations have been solved numerically using fourth order Runge–Kutta–Gill method, for various values of M, f_w, g_w, λ, ω and n. We have taken Prandtl number $Pr = 0.72$, $\beta = 0.5$. The step size of $\Delta\eta = 0.1$ was taken for integration and η^* was taken to be between 4.5 and 6. Convergence is considered to have been achieved if the difference between the values of $f''(0)$ and $g'(0)$ in two successive iterations is less than 10^{-5}.

Although numerical calculations were carried out for 504 conditions involving various values of the parameters, for the sake of brevity, only some representative velocity and enthalpy profiles are presented in Figs. 1–4. From Figs. 1–2, we observe that there is a velocity overshoot when there is a magnetic field. The condition for the occurrence of the velocity overshoot is that $Mn > 1$, $g_w < 1$ [20]. Figure 2 reveals that, for

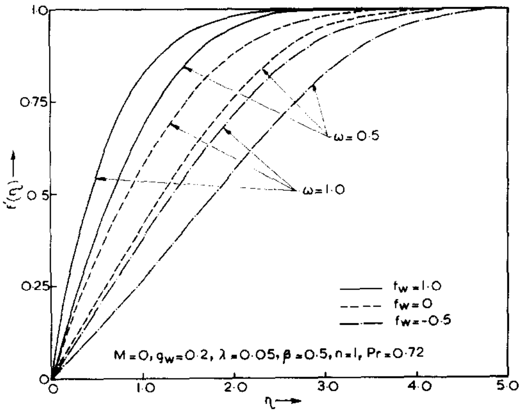

FIG. 1. Velocity profiles ($M = 0$).

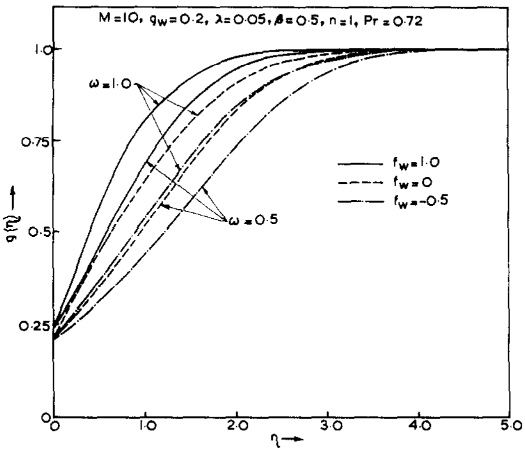

FIG. 4. Total enthalpy profiles ($M = 10$).

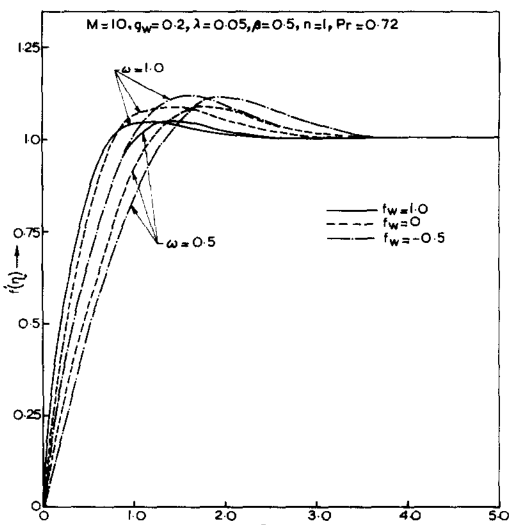

FIG. 2. Velocity profiles ($M = 10$).

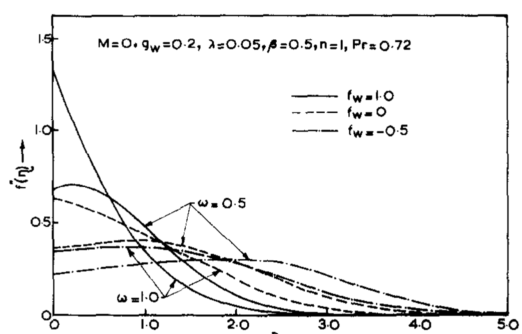

FIG. 5. Shear stress profiles ($M = 0$).

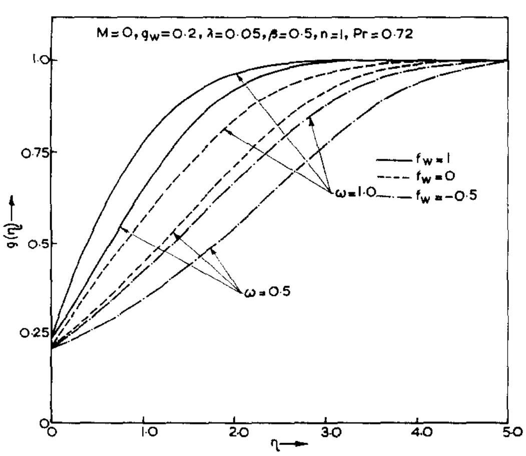

FIG. 3. Total enthalpy profiles ($M = 0$).

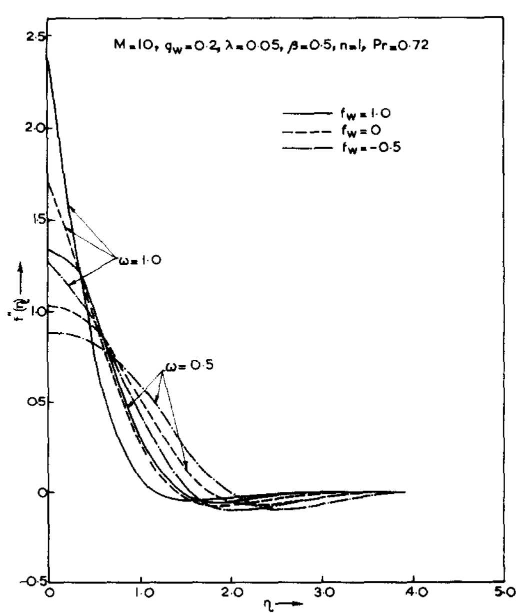

FIG. 6. Shear stress profiles ($M = 10$).

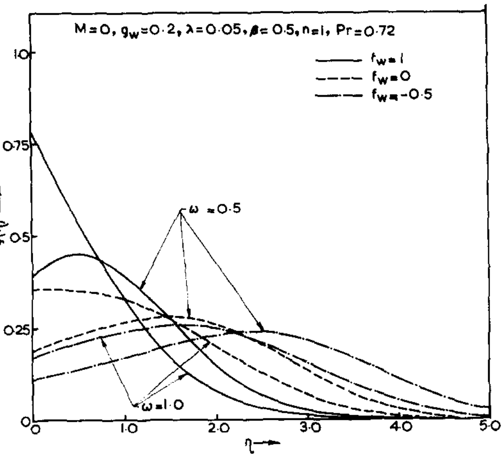

FIG. 7. Heat-transfer profiles ($M = 0$).

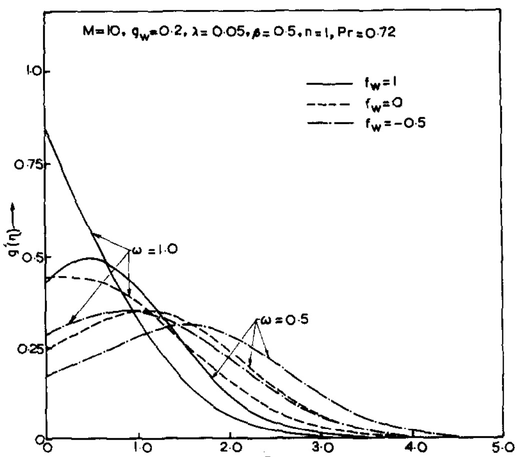

FIG. 8. Heat-transfer profiles ($M = 10$).

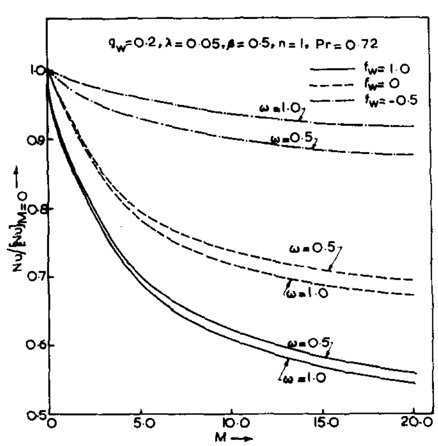

FIG. 9. Nusselt numbers ratio vs magnetic parameter.

given $M \neq 0$ and λ, the effect of suction is to decrease the velocity overshoot, while injection increases it. The effect of decreasing ω from 1 to 0.5 is to make the velocity and total enthalpy profiles less steep, whereas they become more steep when the magnetic parameter M or the suction parameter $f_w(f_w > 0)$ or cooling parameter g_w increases. The effect of the injection parameter $f_w(f_w < 0)$ or the slip parameter λ is contrary to that of M or $f_w(f_w > 0)$. (The profiles for $\lambda = 0$ or $\omega = 0.7$ or $g_w = 0.6$ are not shown in figures for lack of space.)

It is observed from Figs. 5, 7 and 8 that the velocity and enthalpy profiles for $f_w \geq 0$ and $g_w = 0.2$ have a point of inflexion when $\omega = 0.5$, which is not present in $\omega = 1$ profiles as evidenced by a maximum in $f''(\eta)$ and $g'(\eta)$. In the case of injection, the point of inflexion is present both for $\omega = 1$ and $\omega = 0.5$. From the results (graphs not shown for lack of space), we find that for $g_w = 0.6$, the velocity profiles do not possess a point of inflexion, but which continues to be present in the enthalpy profiles. Since the velocity profiles in the presence of the magnetic field (Fig. 2) have a velocity overshoot, there is a point of inflexion in the velocity profiles whatever may be the values of f_w, ω and g_w as is evident from the minimum of $f''(\eta)$ shown in Fig. 6. The above results hold good both for slip and no-slip case. It may be remarked that similar effects have been observed by Gross and Dewey [17] on using the power-law relation for viscosity for two-dimensional stagnation-point flow without magnetic field and slip.

Tables 1–6 show the effect of M, f_w, g_w, λ, n and ω on $f''(0)$ and $g'(0)$, the skin friction and heat transfer parameters respectively. We observe that for fixed values of f_w, M, g_w, n and λ, both $f''(0)$ and $g'(0)$ decrease with ω. The decrease is more pronounced when $g_w = 0.2$ as compared to $g_w = 0.6$. Also we find that injection reduces the skin friction and heat transfer and suction or magnetic field does the reverse. We further observe that increasing n results in higher values of $f''(0)$ and $g'(0)$. For $f_w = 2.0$, $g'(0)$ does not vary much as n is increased from 1 to 10. It is observed that $f''(0)$ is strongly dependent on n. Hence, it is necessary to use the correct value of n for the accurate determination of skin friction and heat-transfer results. On the other hand, when g_w is increased, $f''(0)$ increases, but $g'(0)$ decreases. The results given in Tables 1–3 also reveal that the effect of slip is more predominant for higher values of g_w, M, $f_w(f_w > 0)$ and ω. The skin friction parameter seems to be more affected by slip as compared to the heat-transfer parameter.

For an electrically conducting fluid, the skin-friction coefficient C_f and the Nusselt number Nu, depend upon the velocity gradient in the inviscid flow, which in turn decreases as M increases [19]. Since Nu and C_f respectively are directly and inversely proportional to du_e/dx for given λ, n, f_w and g_w, the combined effect of inviscid and viscous contribution is to decrease the ratio $Nu/[Nu]_{M=0}$ and to increase the ratio $C_f/[C_f]_{M=0}$ when M increases. Figure 9 gives the variation of $Nu/[Nu]_{M=0}$ with M. It is seen that only for injection, the ratio decreases more when $\omega = 0.5$.

7. CONCLUSIONS

The results indicate that there is a reduction in heat-transfer rate (Nusselt number) due to the increase in the magnitude of the magnetic field independent of mass transfer, total enthalpy at the wall, and slip.

Table 1. Skin friction and heat-transfer parameters ($\lambda = 0, n = 1$)
$\beta = 0.5, Pr = 0.72$

M	g_w	ω	$f_w = -0.5$		$f_w = 0$		$f_w = 1.0$		$f_w = 2.0$	
			$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
0	0.2	1.0	0.3395	0.1716	0.6350	0.3545	1.3969	0.8084	2.2742	1.3191
0	0.2	0.7	0.2626	0.1292	0.4400	0.2394	0.8973	0.5138	1.4287	0.8247
0	0.2	0.5	0.2201	0.1065	0.3458	0.1849	0.6704	0.3807	1.0500	0.6036
0	0.6	1.0	0.5078	0.0965	0.7858	0.1842	1.5117	0.4069	2.3629	0.6607
0	0.6	0.7	0.4726	0.0893	0.7085	0.1640	1.3232	0.3535	2.0471	0.5702
0	0.6	0.5	0.4502	0.0847	0.6615	0.1518	1.2115	0.3220	1.8612	0.5169
5	0.2	1.0	0.9655	0.2497	1.3063	0.4184	2.0591	0.8434	2.8712	1.3374
5	0.2	0.7	0.7325	0.1831	0.9308	0.2847	1.3713	0.5396	1.8529	0.8386
5	0.2	0.5	0.6082	0.1486	0.7455	0.2209	1.0518	0.4018	1.3899	0.6152
5	0.6	1.0	1.3556	0.1312	1.6365	0.2131	2.2894	0.4228	3.0357	0.6690
5	0.6	0.7	1.2550	0.1205	1.4906	0.1903	2.0373	0.3684	2.6650	0.5780
5	0.6	0.5	1.1917	0.1138	1.4012	0.1765	1.8864	0.3362	2.4453	0.5244
10	0.2	1.0	1.3948	0.2826	1.7612	0.4479	2.5293	0.8631	3.3249	1.3495
10	0.2	0.7	1.0565	0.2063	1.2673	0.3057	1.7116	0.5542	2.1779	0.8477
10	0.2	0.5	0.8769	0.1669	1.0216	0.2376	1.3276	0.4138	1.6517	0.6229
10	0.6	1.0	1.9141	0.1447	2.2015	0.2256	2.8396	0.4314	3.5484	0.6743
10	0.6	0.7	1.7710	0.1327	2.0111	0.2016	2.5431	0.3763	3.1362	0.5829
10	0.6	0.5	1.6811	0.1252	1.8940	0.1871	2.3649	0.3436	2.8901	0.5292
20	0.2	1.0	2.0578	0.3194	2.4544	0.4818	3.2482	0.8876	4.0363	1.3658
20	0.2	0.7	1.5591	0.2325	1.7842	0.3300	2.2370	0.5724	2.6920	0.8601
20	0.2	0.5	1.2950	0.1879	1.4479	0.2572	1.7562	0.4287	2.0683	0.6332
20	0.6	1.0	2.7559	0.1589	3.0504	0.2390	3.6789	0.4416	4.3546	0.6812
20	0.6	0.7	2.5491	0.1457	2.7943	0.2139	3.3159	0.3857	3.8778	0.5894
20	0.6	0.5	2.4195	0.1374	2.6363	0.1987	3.0965	0.3526	3.5928	0.5354

Table 2. Skin friction and heat-transfer parameters ($\lambda = 0.05, n = 1$)
$\beta = 0.5, Pr = 0.72$

M	g_w	ω	$f_w = -0.5$		$f_w = 0$		$f_w = 1.0$		$f_w = 2.0$	
			$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
0	0.2	1.0	0.3428	0.1746	0.6294	0.3542	1.3242	0.7795	2.0608	1.2292
0	0.2	0.7	0.2660	0.1318	0.4445	0.2436	0.8991	0.5212	1.4175	0.8343
0	0.2	0.5	0.2232	0.1088	0.3516	0.1891	0.6850	0.3928	1.0776	0.6289
0	0.6	1.0	0.5052	0.0982	0.7704	0.1840	1.4244	0.3923	2.1331	0.6156
0	0.6	0.7	0.4707	0.0908	0.6981	0.1645	1.2659	0.3455	1.8954	0.5432
0	0.6	0.5	0.4486	0.0861	0.6537	0.1526	1.1691	0.3170	1.7483	0.4987
5	0.2	1.0	0.9582	0.2538	1.2745	0.4177	1.9297	0.8129	2.5792	1.2460
5	0.2	0.7	0.7348	0.1876	0.9310	0.2905	1.3619	0.5482	1.8242	0.8489
5	0.2	0.5	0.6126	0.1525	0.7525	0.2270	1.0666	0.4156	1.4164	0.6417
5	0.6	1.0	1.3003	0.1333	1.5491	0.2127	2.0980	0.4075	2.6820	0.6232
5	0.6	0.7	1.2091	0.1225	1.4220	0.1908	1.8978	0.3599	2.4157	0.5506
5	0.6	0.5	1.1512	0.1157	1.3428	0.1775	1.7738	0.3309	2.2495	0.5059
10	0.2	1.0	1.3722	0.2871	1.7037	0.4470	2.3527	0.8316	2.9676	1.2569
10	0.2	0.7	1.0541	0.2116	1.2604	0.3123	1.6900	0.5634	2.1323	0.8585
10	0.2	0.5	0.8799	0.1717	1.0267	0.2447	1.3399	0.4285	1.6751	0.6502
10	0.6	1.0	1.8014	0.1468	2.0446	0.2249	2.5570	0.4155	3.0871	0.6279
10	0.6	0.7	1.6766	0.1349	1.8851	0.2020	2.3300	0.3675	2.8007	0.5552
10	0.6	0.5	1.5973	0.1273	1.7851	0.1881	2.1885	0.3383	2.6209	0.5105
20	0.2	1.0	2.0004	0.3243	2.3461	0.4805	2.9883	0.8549	3.5668	1.2717
20	0.2	0.7	1.5434	0.2388	1.7601	0.3376	2.1902	0.5824	2.6135	0.8714
20	0.2	0.5	1.2918	0.1937	1.4460	0.2654	1.7601	0.4447	2.0820	0.6616
20	0.6	1.0	2.5257	0.1611	2.7592	0.2381	3.2303	0.4251	3.7017	0.6341
20	0.6	0.7	2.3549	0.1479	2.5559	0.2143	2.9667	0.3766	3.3867	0.5613
20	0.6	0.5	2.2463	0.1396	2.4276	0.1997	2.8009	0.3471	3.1874	0.5164

Table 3. Skin friction and heat-transfer parameters ($\lambda = 0.1$, $n = 1$)
$\beta = 0.5$, $Pr = 0.72$

M	g_w	ω	$f_w = -0.5$		$f_w = 0$		$f_w = 1.0$		$f_w = 2.0$	
			$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
0	0.2	1.0	0.3455	0.1773	0.6225	0.3532	1.2561	0.7512	1.8806	1.1488
0	0.2	0.7	0.2692	0.1344	0.4482	0.2474	0.8949	0.5253	1.3870	0.8323
0	0.2	0.5	0.2262	0.1109	0.3570	0.1933	0.6959	0.4033	1.0914	0.6472
0	0.6	1.0	0.5019	0.0996	0.7541	0.1833	1.3440	0.3780	1.9405	0.5754
0	0.6	0.7	0.4682	0.0922	0.6868	0.1647	1.2105	0.3369	1.7588	0.5170
0	0.6	0.5	0.4467	0.0874	0.6450	0.1532	1.1266	0.3114	1.6419	0.4798
5	0.2	1.0	0.9483	0.2572	1.2399	0.4158	1.8085	0.7826	2.3321	1.1638
5	0.2	0.7	0.7357	0.1918	0.9282	0.2956	1.3416	0.5527	1.7693	0.8468
5	0.2	0.5	0.6164	0.1564	0.7576	0.2328	1.0742	0.4273	1.4228	0.6607
5	0.6	1.0	1.2470	0.1347	1.4672	0.2113	1.9307	0.3920	2.3952	0.5819
5	0.6	0.7	1.1646	0.1240	1.3567	0.1906	1.7708	0.3506	2.2001	0.5237
5	0.6	0.5	1.1118	0.1173	1.2867	0.1778	1.6689	0.3247	2.0734	0.4865
10	0.2	1.0	1.3455	0.2905	1.6426	0.4443	2.1878	0.8000	2.6653	1.1735
10	0.2	0.7	1.0491	0.2164	1.2487	0.3180	1.6542	0.5681	2.0552	0.8563
10	0.2	0.5	0.8814	0.1764	1.0288	0.2514	1.3420	0.4408	1.6730	0.6696
10	0.6	1.0	1.6973	0.1482	1.9034	0.2231	2.3176	0.3994	2.7219	0.5861
10	0.6	0.7	1.5887	0.1364	1.7697	0.2017	2.1422	0.3577	2.5181	0.5278
10	0.6	0.5	1.5189	0.1289	1.6844	0.1883	2.0295	0.3317	2.3850	0.4907
20	0.2	1.0	1.9367	0.3275	2.2339	0.4769	2.7475	0.8215	3.1706	1.1865
20	0.2	0.7	1.5230	0.2444	1.7280	0.3438	2.1234	0.5873	2.4950	0.8691
20	0.2	0.5	1.2858	0.1993	1.4387	0.2730	1.7488	0.4578	2.0616	0.6816
20	0.6	1.0	2.3246	0.1621	2.5103	0.2357	2.8668	0.4081	3.2033	0.5914
20	0.6	0.7	2.1831	0.1492	2.3479	0.2134	2.6723	0.3662	2.9887	0.5332
20	0.6	0.5	2.0918	0.1410	2.2434	0.1996	2.5461	0.3400	2.8472	0.4961

Table 4. Skin friction and heat-transfer parameters ($\lambda = 0$, $n = 3$)
$\beta = 0.5$, $Pr = 0.72$

M	g_w	f_w	$\omega = 1.0$		$\omega = 0.7$		$\omega = 0.5$	
			$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
5	0.2	−0.5	1.1445	0.2770	0.8596	0.2014	0.7091	0.1626
5	0.2	0.0	1.5406	0.4458	1.0902	0.3029	0.8689	0.2348
5	0.2	1.0	2.3584	0.8633	1.5691	0.5534	1.2021	0.4126
5	0.2	2.0	3.1815	1.3496	2.0569	0.8473	1.5445	0.6222
5	0.6	−0.5	1.5532	0.1429	1.4360	0.1310	1.3624	0.1235
5	0.6	0.0	1.8632	0.2240	1.6956	0.2001	1.5929	0.1856
5	0.6	1.0	2.5394	0.4301	2.2611	0.3750	2.0945	0.3423
5	0.6	2.0	3.2768	0.6732	2.8804	0.5819	2.6452	0.5281
10	0.2	−0.5	1.7394	0.3240	1.3008	0.2340	1.0708	0.1882
10	0.2	0.0	2.1916	0.4893	1.5613	0.3334	1.2500	0.2589
10	0.2	1.0	3.0655	0.8948	2.0681	0.5761	1.5996	0.4309
10	0.2	2.0	3.8864	1.3700	2.5490	0.8624	1.9338	0.6346
10	0.6	−0.5	2.2813	0.1622	2.1075	0.1483	1.9985	0.1397
10	0.6	0.0	2.6092	0.2419	2.3808	0.2162	2.2404	0.2007
10	0.6	1.0	3.2820	0.4428	2.9406	0.3867	2.7351	0.3534
10	0.6	2.0	3.9803	0.6813	3.5229	0.5894	3.2505	0.5353
20	0.2	−0.5	2.7089	0.3799	2.0208	0.2731	1.6618	0.2190
20	0.2	0.0	3.2317	0.5420	2.3189	0.3704	1.8651	0.2882
20	0.2	1.0	4.1865	0.9355	2.8660	0.6055	2.2389	0.4546
20	0.2	2.0	5.0220	1.3984	3.3482	0.8834	2.5697	0.6518
20	0.6	−0.5	3.4199	0.1837	3.1581	0.1678	2.9943	0.1580
20	0.6	0.0	3.7664	0.2622	3.4455	0.2347	3.2477	0.2180
20	0.6	1.0	4.4392	0.4585	4.0016	0.4011	3.7370	0.3671
20	0.6	2.0	5.1011	0.6921	4.5489	0.5995	4.2182	0.5449

Table 5. Skin friction and heat-transfer parameters ($\lambda = 0$, $n = 5$)

$\beta = 0.5$, $Pr = 0.72$

M	g_w	f_w	$\omega = 1.0$		$\omega = 0.7$		$\omega = 0.5$	
			$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
5	0.2	−0.5	1.2245	0.2900	0.9150	0.2100	0.7524	0.1691
5	0.2	0.0	1.6564	0.4599	1.1671	0.3124	0.9277	0.2421
5	0.2	1.0	2.5290	0.8753	1.6797	0.5621	1.2854	0.4197
5	0.2	2.0	3.3737	1.3577	2.0423	0.5849	1.6390	0.6281
5	0.6	−0.5	1.6810	0.1506	1.5525	0.1378	1.4719	0.1298
5	0.6	0.0	2.0174	0.2315	1.8341	0.2067	1.7219	0.1917
5	0.6	1.0	2.7216	0.4356	2.4231	0.3799	2.2443	0.3470
5	0.6	2.0	3.4593	0.6766	2.1822	0.8539	2.7949	0.5310
10	0.2	−0.5	1.8983	0.3440	1.4102	0.2472	1.1561	0.1982
10	0.2	0.0	2.4082	0.5109	1.7054	0.3477	1.3599	0.2698
10	0.2	1.0	3.3739	0.9138	2.2679	0.5895	1.7498	0.4415
10	0.2	2.0	4.2374	1.3836	2.7765	0.8728	2.1047	0.6434
10	0.6	−0.5	2.5305	0.1739	2.3346	0.1588	2.2121	0.1494
10	0.6	0.0	2.8975	0.2533	2.6405	0.2264	2.4826	0.2101
10	0.6	1.0	3.6136	0.4515	3.2362	0.3945	3.0090	0.3608
10	0.6	2.0	4.3144	0.6870	3.8201	0.5946	3.5255	0.5402
20	0.2	−0.5	3.0199	0.4095	2.2342	0.2926	1.8278	0.2338
20	0.2	0.0	3.6331	0.5735	2.5863	0.3913	2.0694	0.3041
20	0.2	1.0	4.7347	0.9643	3.2222	0.6253	2.5068	0.4701
20	0.2	2.0	5.6470	1.4202	3.7534	0.8994	2.8738	0.6650
20	0.6	−0.5	3.8965	0.2009	3.5929	0.1832	3.4032	0.1722
20	0.6	0.0	4.2975	0.2790	3.9253	0.2497	3.6962	0.2319
20	0.6	1.0	5.0309	0.4717	4.5314	0.4130	4.2292	0.3782
20	0.6	2.0	5.6969	0.7011	5.0809	0.6077	4.7118	0.5527

Table 6. Skin friction and heat-transfer parameters

($\lambda = 0$, $n = 10$)

$\beta = 0.5$, $Pr = 0.72$

M	g_w	f_w	$\omega = 1.0$		$\omega = 0.7$		$\omega = 0.5$	
			$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
5	0.2	−0.5	1.3081	0.3044	0.9718	0.2193	0.7962	0.1761
5	0.2	0.0	1.7871	0.4767	1.2518	0.3232	0.9912	0.2501
5	0.2	1.0	2.7484	0.8914	1.8172	0.5727	1.3860	0.4276
5	0.2	2.0	3.6452	1.3697	2.3517	0.8620	1.7627	0.6343
5	0.6	−0.5	1.8412	0.1609	1.6975	0.1470	1.6076	0.1383
5	0.6	0.0	2.2276	0.2424	2.0212	0.2163	1.8950	0.2006
5	0.6	1.0	3.0018	0.4445	2.6693	0.3879	2.4703	0.3543
5	0.6	2.0	3.7603	0.6825	3.3066	0.5903	3.0371	0.5360
10	0.2	−0.5	2.0663	0.3660	1.5234	0.2615	1.2430	0.2089
10	0.2	0.0	2.6534	0.5361	1.8642	0.3639	1.4788	0.2818
10	0.2	1.0	3.7678	0.9390	2.5148	0.6059	1.9307	0.4538
10	0.2	2.0	4.7293	1.4035	3.0837	0.8861	2.3289	0.6536
10	0.6	−0.5	2.8524	0.1899	2.6258	0.1730	2.4843	0.1625
10	0.6	0.0	3.2987	0.2700	2.9981	0.2411	2.8140	0.2237
10	0.6	1.0	4.1290	0.4657	3.6903	0.4072	3.4264	0.3725
10	0.6	2.0	4.8705	0.6969	4.3094	0.6035	3.9747	0.5485
20	0.2	−0.5	3.3532	0.4422	2.4574	0.3137	1.9985	0.2496
20	0.2	0.0	4.0900	0.6103	2.8824	0.4149	2.2911	0.3216
20	0.2	1.0	5.4324	1.0018	3.6608	0.6496	2.8285	0.4883
20	0.2	2.0	6.5185	1.4515	4.2981	0.9203	3.2715	0.6809
20	0.6	−0.5	4.5356	0.2249	4.1708	0.2044	3.9436	0.1918
20	0.6	0.0	5.0573	0.3037	4.6043	0.2715	4.3262	0.2520
20	0.6	1.0	5.9676	0.4932	5.3595	0.4322	4.9922	0.3960
20	0.6	2.0	6.7051	0.7170	5.9701	0.6221	5.5298	0.5661

Again for a given M, heat-transfer rate is reduced due to injection or due to increase in the total enthalpy at the wall. It has been found that the skin friction and heat-transfer parameters are affected appreciably with the variations of ω, the index in the power-law, only at low wall temperature, their variations with ω being considerably small at high values of the wall temperature. Therefore, it can be concluded that the linear viscosity–temperature relation is not a good approximation at low wall temperatures for obtaining skin friction and heat-transfer results. It has been also observed that the variation of the density–viscosity product (i.e. $\omega \neq 1$) across the boundary layer gives rise to a point of inflexion in the velocity and enthalpy profiles. In the presence of magnetic field for all values of ω, the velocity profiles exhibit velocity overshoot in a certain region and the suction parameter tends to reduce the magnitude of the velocity overshoot. The present analysis also shows that the quasi-linearization technique is another powerful tool for solving nonlinear two-point boundary value problems and the computer time required for the solution is much less compared to that for other numerical methods.

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ECOULEMENT HYPERSONIQUE D'UN GAZ RAREFIE MAGNETIQUE AU POINT D'ARRET D'UN SOLIDE EMOUSSE AVEC TRANSITION ENTRE REGIMES MOLECULAIRE ET LAMINAIRE ET TRANSFERT DE MASSE

Résumé—L'écoulement visqueux hypersonique d'un gaz légèrement raréfié, conducteur de l'électricité et avec des propriétés variables, est étudié dans la zone du point d'arrêt d'un solide émoussé, en présence d'un champ magnétique, en tenant compte de la transition entre régimes moléculaire et laminaire et du transfert de masse (aspiration ou injection). Des solutions en similitude des équations de la couche-limite ont été obtenues numériquement à l'aide de la méthode de quasi-linéarisation. Les résultats indiquent dans quelle mesure le taux de transfert thermique se trouve affecté par les paramètres caractérisant, le champ magnétique, le glissement et le transfert de masse.

DIE MAGNETISCHE, HYPERSONISCHE STRÖMUNG EINES VERDÜNNTEN GASES AM STAUPUNKT EINES STUMPFEN KÖRPERS MIT IMPULS- UND MASSENAUSTAUSCH

Zusammenfassung—Es wurde die hypersonische, zähe Strömung eines geringfügig verdünnten, elektrisch leitfähigen Gases mit veränderlichen Stoffwerten im Staubereich eines stumpfen Körpers in einem Magnetfeld und der Impuls- und Massenaustausch (Absaugung oder Einspritzung) untersucht. Die auf der Ähnlichkeitstheorie beruhenden Lösungen der Grenzschichtgleichungen wurden numerisch mit Hilfe der Methode der Quasi-Linarisierung gewonnen. Die Ergebnisse zeigen das Ausmaß der Beeinflußung des Wärmeübergangs durch das Magnetfeld, die Impuls- und Massenaustauschparameter u.a.

МАГНИТНЫЙ ГИПЕРЗВУКОВОЙ ПОТОК РАЗРЕЖЕННОГО ГАЗА В КРИТИЧЕСКОЙ ТОЧКЕ ЗАТУПЛЕННОГО ТЕЛА ПРИ НАЛИЧИИ СКОЛЬЖЕНИЯ И ПЕРЕНОСА МАССЫ

Аннотация — Исследуется гиперзвуковой вязкий поток слегка разреженного электропроводящего газа с переменными свойствами в критической области затупленного тела при наличии магнитного поля, скольжения и переноса массы (отсоса или вдува). С помощью метода квазилинеаризации получены численные автомодельные решения уравнений пограничного слоя. Результаты показывают степень влияния параметров магнитного поля, скольжения и переноса массы на скорость переноса тепла.